

Complexity Olympics — Station Cards

Laminate and place at each station before the lesson.

STATION 1: $O(1)$ — CONSTANT TIME

Event: Direct Lookup

Your task: Someone calls out a position number. Find that card directly.

Setup: Take a stack of numbered cards, laid out in order.

Procedure:

1. Someone calls out a number (e.g., "card 7")
2. Go directly to that position. Pick it up.
3. Count the steps: always **1 step**, no matter how many cards there are.

Try it for:

- Stack of 10 cards → steps needed: ____
- Stack of 100 cards → steps needed: ____
- Stack of 1,000 cards → steps needed: ____

Record on your worksheet!

The Point: This is what an array index does — `cards[7]` retrieves the value in exactly 1 step, whether the array has 10 or 10,000 items. A dictionary key lookup is also $O(1)$.

STATION 2: $O(\log n)$ — LOGARITHMIC TIME

Event: Binary Search

Your task: Find a target number in a sorted stack using binary search.

Target number is on the sticky note at this station.

Procedure:

1. Lay out your sorted cards in order.
2. Always check the MIDDLE card.
3. If middle = target: done!
4. If target is higher: take the top half. Repeat.
5. If target is lower: take the bottom half. Repeat.
6. Count every time you check a card.

Try it for:

- 8 cards (target = 6th card) → guesses: ____

- 16 cards (target = 11th card) → guesses: ____
- 32 cards (target = 25th card) → guesses: ____

The Point: Every time you double the input, you only need ONE more guess. $\log_2(8)=3$, $\log_2(16)=4$, $\log_2(32)=5$.

STATION 3: $O(n)$ — LINEAR TIME

Event: Finding the Maximum

Your task: Find the highest-valued card in a shuffled pile.

Procedure:

1. Shuffle your cards face-down.
2. Turn them over one at a time.
3. Keep track of the highest value seen so far.
4. When you've looked at every card, you've found the maximum.
5. Count how many cards you looked at.

Try it for:

- $n=5$ cards → cards looked at: ____
- $n=10$ cards → cards looked at: ____
- $n=20$ cards → cards looked at: ____

The Point: You must look at every card at least once. Double the input = double the work. There's no shortcut — you can't skip any card when finding a maximum in an unsorted collection.

STATION 4: $O(n^2)$ — QUADRATIC TIME

Event: Finding All Duplicate Pairs

Your task: Check every card against every other card.

Procedure:

1. Take n cards.
2. Compare card 1 to card 2, card 3, card 4... ($n-1$ comparisons)
3. Compare card 2 to card 3, card 4... ($n-2$ comparisons)
4. Continue until all pairs checked.
5. Count every comparison.

Prediction formula: comparisons = $n \times (n-1) \div 2$

Try it for:

- $n=3$ → predicted: ____, actual: ____
- $n=4$ → predicted: ____, actual: ____
- $n=5$ → predicted: ____, actual: ____

- $n=6 \rightarrow$ predicted: ____, actual: ____

Warning: For $n=10$, this is 45 comparisons. For $n=100$: 4,950. For $n=1,000$: 499,500.

The Point: Double the input = FOUR TIMES the work. This grows very fast — quadratic growth is impractical for large datasets.